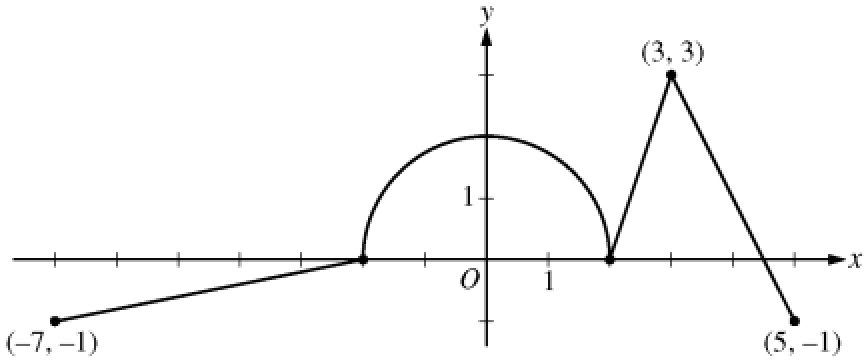
Analysis Homework

1.



Graph of $g'$

The function $g$ is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of $g$, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find $g(3)$ and $g(-2)$.

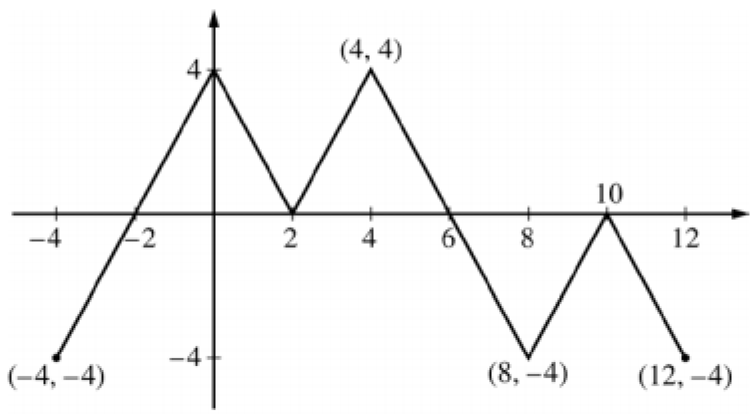
(b) Find the $x$-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.

(c) The function $h$ is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the $x$-coordinate of each critical point of $h$, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

2.

The figure above shows the graph of the piecewise-linear function $f$. For $-4 \le x \le 12$, the function $g$ is defined by

$g(x) = \int_2^x f(t)\,dt$.



Graph of $f$

(a) Does $g$ have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

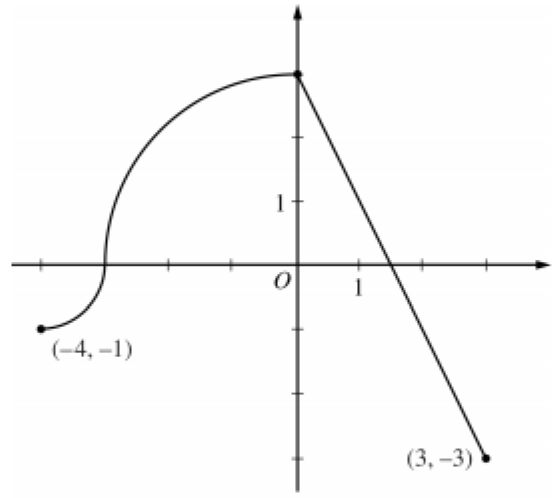
(b) Does the graph of $g$ have a point of inflection at $x = 4$? Justify your answer.

(c) Find the absolute minimum value and the absolute maximum value of $g$ on the interval $-4 \le x \le 12$. Justify your answers.

(d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.

3.

The continuous function $f$ is defined on the interval $-4 \le x \le 3$. The graph of $f$ consists of two quarter circles and one line segment, as shown in the figure above.



Graph of $f$

Let $g(x) = 2x + \int_0^x f(t)\,dt$.

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

(b) Determine the $x$-coordinate of the point at which $g$ has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

(c) Find all values of $x$ on the interval $-4 < x < 3$ for which the graph of $g$ has a point of inflection. Give a reason for your answer.

(d) Find the average rate of change of $f$ on the interval $-4 \le x \le 3$. There is no point $c$, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.